

On the Cauchy problem for the generalized Boussinesq equation with logarithmic nonlinearity

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Abstract

This study examines the global existence and finite-time blow-up of solutions to the following Cauchy problem of the one-dimensional fifth-order Boussinesq equation with logarithmic nonlinearity.

$$u_{tt} - u_{xx} - u_{xxtt} + (u_{xx} + f(u))_{xx} - \alpha u_{xxt} + u_{xxxxt} = 0, \quad x \in \mathbb{R}, \quad t > 0, \quad (1)$$

subject to initial conditions values

$$u(x, 0) = u_0(x), \quad u_t(x, 0) = u_1(x), \quad x \in \mathbb{R}, \quad (2)$$

where $u = u(x, t)$ is the unknown real valued function of $x \in \mathbb{R}$ and $t > 0$, $f(u) = |u|^{p-2} u \ln |u|$, ($p > 2$) represents the nonlinearity, $\alpha > 0$ is a constant, and $u_0(x)$ and $u_1(x)$ are given initial data. Firstly, we demonstrate the existence and uniqueness of local mild solutions in the energy space through the application of the contraction mapping principle. Furthermore, under certain constraints on the initial data, we present the findings on the existence and uniqueness of global solutions and finite-time blow-up of solutions through the utilisation of the potential well method. Additionally, we provide the sufficient and necessary conditions for global existence and finite-time blow-up of solutions.

Keywords: Cauchy problem, stable set and unstable set, existence of global solution, nonexistence.

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