

# Vortex patch problem for two-dimensional partial viscous Boussinesq equations

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## Abstract

This work addresses local well-posedness for two-dimensional Boussinesq equations with zero viscosity as soon as the initial vortex is a smooth patch. We demonstrate that if the initial vorticity is a regular patch, specifically a characteristic function of a bounded domain  $\Omega$  in  $\mathbb{R}^2$ , where the boundary  $\partial\Omega$  is a Jordan curve with Hölder regularity, and the initial density  $\rho$  is a smooth function, then the associated velocity vector field  $v$  is a locally Lipschitz function. Furthermore, the transported vorticity by the flow  $\psi$  associated with the velocity keeps its initial regularity through the time. We provide also the rate convergence in Lebesgue spaces  $L^p$  with  $2 \leq p \leq \infty$  towards the two-dimensional inviscid Boussinesq equations when the diffusion parameter  $\kappa$  goes to zero.

**Keywords:** Partial viscous Boussinesq equations, Vortex patch initial data, Inviscid limit.

**AMS Classification :** 76D03, 35B30.

## References:

- [1] J.-Y. Chemin: *Perfect incompressible Fluids*. Oxford University Press (1998).
- [2] R. Danchin: *Poches de tourbillon visqueuses*. J. Math. Pures Appl. (9) 76, No. 7, 609–647 (1997).
- [3] T. Hmidi: *Régularité höldérienne des poches de tourbillon visqueuses*. J. Math. Pures Appl. (9) 84, No. 11, 1455-1495 (2005).
- [4] T. Hmidi and M. Zerguine: *Vortex patch problem for stratified Euler equations*. Comm. Math. Sci. Vol. 12, No. 8, 1541–1563 (2014).
- [5] H. Meddour: *Local persistence of geometric structure for Boussinesq system with zero viscosity*. Mathematicki Vesnik, Vol. 71, No. 4, 285–303 (2019).
- [6] V. I. Yudovich: *Non-stationary flows of an ideal incompressible fluid*. Zhurnal Vych Matematika, 3, 1032–106 (1963).